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# 一种二元响应变量模型的分布式贝叶斯估计方法

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**摘要**:在海量数据背景下,传统的基于单个计算节点的算法很难满足分析要求。考察了一种分布式贝叶斯估计方法,通过在每台机器上单独运行蒙特卡洛抽样并做加权平均可以有效地解决算法效率问题。将该方法应用于基于广义极值模型的二元响应变量回归分析,并探讨其实用性。模拟研究表明分布式算法比传统方法更有效。

**关键词**:海量数据; 分布式贝叶斯方法; 极值模型
**中图分类号**:TP301.6 **文献标志码**:A

# A Distributed Bayesian Regression Method for Binary Response Massive Data

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**Abstract**: In the background of massive data, it is difficult to meet the analysis requirements for traditional one-node based algorithm. This paper considers a distributed Bayesian estimation method to solve the GEV based general linear regression model by running a separate Monte Carlo algorithm on each machine. The method is applied to regression analysis of binary response variables based on generalized extreme value model. The results show that the proposed distributed Bayesian regression algorithm is much faster than the traditional algorithm in the simulated data sets studying.

**Key words**: massive data; distributed Bayesian regression; GEV model

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0-1 。 ,

, 0-1 。 , Wang [2]

, (GEV)

, , GEV

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[1]。 ,

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(MCMC) ,

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[3] 。

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## 1

(Be)

$\boldsymbol{Y}=(Y_1,Y_2,\cdots,Y_n)^{\mathrm{T}}$ $n$

0-1 $,\boldsymbol{X}=(\boldsymbol{x}_1,\boldsymbol{x}_2,\cdots,\boldsymbol{x}_n)^{\mathrm{T}}$

,

$\boldsymbol{x}_i=(x_{i1},x_{i2},\cdots,x_{ik})$, $\boldsymbol{\beta}_c=(\beta_1,\beta_2,\cdots,\beta_k)^{\mathrm{T}}$ $k$ 。 ,

$Y_i$ , $Y_i=1$

$p_i$, $Y_i=0$ $1-p_i$。 ,

$$p_i=Pr(Y_i=1\mid\boldsymbol{\beta}_c,\boldsymbol{x}_i)=\Psi(\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{\beta}_c)$$

$\Psi(\cdot)$ 0 1 ,

, $\Psi(\cdot)$ , $\Psi^{-1}(\cdot)$

。

logit

$\Psi^{-1}(p_i)=\lg\{p_i/(1-p_i)\}$, probit $\Psi^{-1}(p_i)=\Phi^{-1}(p_i)$,$\Phi^{-1}$ ,

cloglog $\Psi^{-1}(p_i)=-\lg\{-\lg(p_i)\}$ 。

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Wang [2]

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, :

$$G(x)=\exp\left[-\left\{1+\xi\frac{(x-\mu)}{\sigma}\right\}_+^{-1/\xi}\right]$$

$\xi$ ,

。 $\xi$ , ,

,

[4] 。 MCMC

, Metropolis-Hastings

(M-H) :

Step 0 $\boldsymbol{\theta}_0$;

Step 1

$q(\boldsymbol{\theta}^{(s-1)},\boldsymbol{\theta}^*)$ $\boldsymbol{\theta}^*$;

Step 2

$\alpha(\boldsymbol{\theta}^{(s-1)},\boldsymbol{\theta}^*)$;

Step 3 $\alpha(\boldsymbol{\theta}^{(s-1)},\boldsymbol{\theta}^*)$ $\boldsymbol{\theta}^{(s)}=\boldsymbol{\theta}^*$,

$1-\alpha(\boldsymbol{\theta}^{(s-1)},\boldsymbol{\theta}^*)$ $\boldsymbol{\theta}^{(s)}=\boldsymbol{\theta}^{(s-1)}$;

Step 4 Step 1,Step 2 Step 3 $S$ ;

Step 5 $\boldsymbol{\theta}_0,\boldsymbol{\theta}_1,\cdots,\boldsymbol{\theta}_S$ $\boldsymbol{\theta}$

, $\boldsymbol{\theta}$ 。

$$\alpha(\boldsymbol{\theta}^{(s-1)},\boldsymbol{\theta}^*)=\min\left[\frac{p(\boldsymbol{\theta}=\boldsymbol{\theta}^*\mid\boldsymbol{y})q(\boldsymbol{\theta}^*;\boldsymbol{\theta}=\boldsymbol{\theta}^{(s-1)})}{p(\boldsymbol{\theta}=\boldsymbol{\theta}^{(s-1)}\mid\boldsymbol{y})q(\boldsymbol{\theta}^{(s-1)};\boldsymbol{\theta}=\boldsymbol{\theta}^*)},1\right]$$

, $p(\boldsymbol{\theta}=\boldsymbol{\theta}^*\mid y)$ $\boldsymbol{\theta}=\boldsymbol{\theta}^*$

, $q(\boldsymbol{\theta}^*;\boldsymbol{\theta}=\boldsymbol{\theta}^{(s-1)})$ $\boldsymbol{\theta}$

$\boldsymbol{\theta}=\boldsymbol{\theta}^{(s-1)}$ 。

## 2 (DBe)

### 2.1 模型参数估计

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:

$\boldsymbol{y}$ ( $n$), $\boldsymbol{y}_s$ $s$

( $n_s$), $\boldsymbol{\theta}$ 。

, :

$$p(\boldsymbol{\theta} \mid \boldsymbol{y}) \propto \prod_{s=1}^{S} p(\boldsymbol{y}_s \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})^{1/S} \quad (1)$$

(1) ,

$S$ ,

。 ,

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。 , :

$$p(\boldsymbol{y}_s \mid \boldsymbol{\theta}) = \prod_{i=1}^{n_s} y_i F_{\mathrm{GEV}}(\beta_0 + \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{\beta}_c; \xi) + (1 - y_i) \cdot [1 - F_{\mathrm{GEV}}(\beta_0 + \boldsymbol{x}_i^{\mathrm{T}} \boldsymbol{\beta}_c; \xi)]$$

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,

Metropolis-Hastings ,

,

[5]。

$s$ $G$

Metropolis-Hastings ,

$S$

:

$\boldsymbol{\theta}_g = \left(\sum_{s=1}^{S} \boldsymbol{W}_s\right)^{-1} \sum_{s=1}^{S} \boldsymbol{W}_s \boldsymbol{\theta}_{sg}$, $g = 1, 2, \cdots, G$

, $\boldsymbol{W}_s$ ,

$\boldsymbol{\theta}$ 。DBe

:

Step 1 $\boldsymbol{y}$ $\boldsymbol{y}_1, \cdots, \boldsymbol{y}_s$;

Step 2 $p(\boldsymbol{\theta})^{1/S}$, $S$

M-H ( ) $\boldsymbol{\theta}_{sg} \sim p(\boldsymbol{\theta} \mid \boldsymbol{y}_s)$, $g = 1, \cdots, G$;

Step 3 $\boldsymbol{\theta}_{sg}$, $s = 1, \cdots, S; g = 1, \cdots, G$ ,

$\boldsymbol{\theta}_g$, $g = 1, \cdots, G$;

Step 4 $\alpha$,

$(1-\alpha)\,G$ ;

## 2.2 其他非参数估计策略

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。 ( 1),

(Kernel density estimate)

$p(\hat{\boldsymbol{\theta}} \mid \boldsymbol{y}_s)$, $p(\hat{\boldsymbol{\theta}} \mid \boldsymbol{y}) \propto \prod_{s=1}^{S} p(\hat{\boldsymbol{y}}_s \mid \boldsymbol{\theta}) p(\boldsymbol{\theta})^{1/S}$。

, $\boldsymbol{\theta}$ ,

[3]。

# 3

$n = 100\,000$, 3 ,

0、 1 ,

$\beta_0 = 1, \beta_1 = 2, \beta_2 = 3, \beta_3 = 4$,

$\xi = 2$, 50 。

, $S = 12, G = 1\,250, \alpha = 0.8$。

: Hadoop2. x: 1

Master 3 Slave , CPU

Intel(R) Xeon(R) CPU E5-2620 0 @ 2.00 GHz * 8 * (1[M]+3[S]) = 32core, 32 (1[M]+3[S]) = 128 GB, JVM -Xmx768 M,

Container 1 GB[6]。

1

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3 。 ,

Hadoop Map ( 12 ) ,

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, , M-H

0, 0.005 0.1

,50

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1 。

表 1

**Table 1** Comparative analysis of distributed Bayes vs. classical method

| MH Sampling | CPU Time/s | $\xi$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ |
|---|---|---|---|---|---|---|
| DBe | 46 | 1.924 6 (0.066 9) | 0.938 8 (0.036 0) | 1.731 8 (0.044 9) | 2.616 3 (0.069 5) | 3.488 3 (0.091 5) |
| Be | 124 | 1.925 7 (0.050 8) | 0.947 7 (0.036 4) | 1.741 8 (0.044 1) | 2.632 2 (0.068 2) | 3.510 1 (0.088 3) |

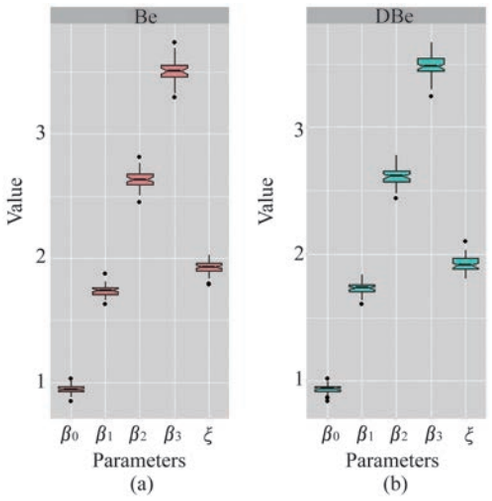


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Fig. 1 Comparative analysis of distributed Bayes vs. classical method

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(Metropolish-Hastings )

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Gibbs

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